

Roll No.

MCSE/MSE-101**M.E./M.Tech., I Semester**

Examination, June 2023

Advanced Computational Mathematics*Time : Three Hours**Maximum Marks : 70***Note:** i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Define linearly dependent and independent sets. Check whether the vectors are $a_1 = (1, 2, 3)$, $a_2 = (1, 0, 1)$ and $a_3 = (0, 1, 0)$ are linearly independent or linearly dependent

b) Define each of the following.

i) Hash function

ii) Heaviside's unit function and error function

iii) Modular arithmetic

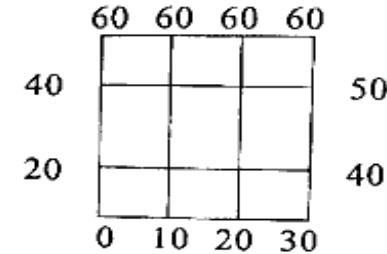
2. a) Prove that $H_n(-x) = (-1)^n H_n(x)$

b) Define $T: V_3 \rightarrow V_2$ by the rule

$$T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_2)$$

Show that this is a linear transformation.

3. a) Find the Fourier transform of $e^{-|x|}$.
- b) Solve the Laplace equation $u_{xx} + u_{yy} = 0$, for the mesh with boundary values shown in the following figure.



4. a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$ subject to initial condition $u = \sin \pi x$ at $t = 0$ for $0 \leq x \leq 1$ and $u = 0$ at $x = 0$ and $x = 1$ for $t > 0$, by Gauss-Seidel iterative method.
- b) Write the properties of DFT, WFT and Haar transform.
5. a) Out of 800 families with four children each, how many families would be expected to have
- 2 boys and 2 girls
 - at least one boy
 - No girl
 - at most two girls
- Assume equal probabilities for boys and girls
- b) Find the mean and variance of Poisson's distribution.

6. a) Obtain the steady state difference equation for the queuing model $\{M/M/1: (N/FCFS)\}$ and show that

$$P_n = \frac{(1-\rho)}{1-\rho^{N+1}} \rho^n; 0 \leq n \leq N.$$

- b) Show that normal distribution as the limiting case of Binomial distribution when $p = q$.

7. a) Explain the Markov Chain. Draw transition diagram and write down the properties of Markov Chain.

- b) Customers at a box office window, being managed by a single man, arrive according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 2 minutes. Find the average waiting time of customers.

8. a) Let A and B be fuzzy sets defined on a universal set X.

$$\text{Then prove that: } |A| + |B| = |A \cup B| + |A \cap B|$$

- b) Write the MATLAB statements required to calculate $y(t)$ from the equation

$$y(t) = \begin{cases} -3t^2 + 5, & t \geq 0 \\ 5t + 2, & t < 0 \end{cases}$$

for values of t between -9 and 9 in step of 0.5 .
